1.1 Permutations

Problems Worksheet



1. Evaluate the following.

a. $\frac{3! \times 4!}{6!}$

b. $\frac{201! \times 7 \times 4!}{7! \times 200!}$

$$C. \quad \frac{4! \times 3! \times 2! \times 1! \times 0!}{6! \times 5}$$

- $\mathsf{d.} \quad \frac{13! \times 8! \times 5! \times 4!}{11! \times 9! \times 9!}$
- 2. Simplify the following, writing your answer without factorial notation.

a.
$$\frac{a!}{(a-2)!}$$

b. $\frac{b! \times (b+2)!}{((b-1)!)^2}$

C.
$$\frac{(c+3)(c^2-1)!}{(c^2-9)!}$$

d. $\frac{(d+1)!(d-2)!}{(d!)^2}$

- 3. Complete the following with the aid of a calculator.
 - a. How many unique ways can a child arrange her five dolls on a shelf?
 - b. A child is investigating all the different ways he can arrange a standard deck of cards (52 cards in a deck). Assuming he was doing this very, very quickly, at a rate of 1000 permutations per second, how long would it take for him to view all possible permutations? Give your answer in years and to three significant figures.

c. In order to transfer funds out of a BitCoin wallet, one needs to know the private key that corresponds to that wallet. These keys are 32 characters in length and consist of a random generation of the letters A to Z and numbers 0 to 9. They are not case sensitive. Suppose a 3 GHz computer is programmed to "brute-force" guess a private key. This corresponds to approximately 375 000 000 attempts per second. How long would it take to attempt all possible passwords? Give your answer in years and to three significant figures

- 4. Complete the following with the aid of a calculator. A student is tinkering with the settings of a random password generator. In each question below, repeated characters are possible.
 - a. With a six-character password taken from the letters A to Z in upper and lower case, and the numbers 0 to 9, how many contain the word "Perth" using any combination of upper and lower case letters (e.g. pErTH and PERth are two acceptable examples).

b. A six-character password taken from the letters A to Z in lower case, with a maximum of one unique vowel and a maximum of two vowels present.

- 5. Complete the following with the aid of a calculator.
 - a. Evaluate ${}^{6}P_{3}$ and state its meaning.
 - b. Eight runners are competing in a race and a gold, silver and bronze medal will be awarded. In how many ways can these prizes be awarded, assuming no athletes draw?
 - c. Seven students (five boys and two girls) are to be arranged in a row for a photo.
 - i. In how many ways can they be arranged?
 - ii. In how many arrangements are the two girls next to one another?
 - iii. In how many arrangements are the two girls not next to one another? State the principle you used to determine this value.

- 6. For each word or set of letters below, calculate the number of unique permutations which can be made. A calculator is permitted.
 - a. SMEETS

b. ASSESSEES

c. KANAZAWA

d. HAPPENCHANCE